Physics of switches II

Luca Gammaitoni
NiPS Laboratory

The Physics of realistic switches

In order to be closer to a reasonable physical model, we need to introduce, together with a friction also a fluctuating force and thus a corresponding Langevin equation:

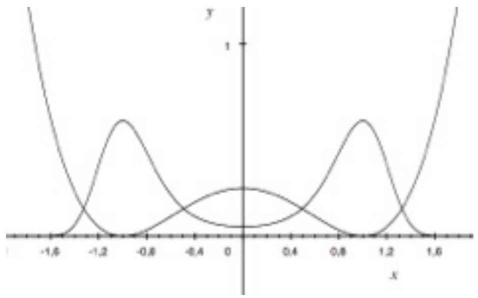
$$m\ddot{x} = -\frac{d}{dx}U(x) - m\gamma\dot{x} + \xi(t) + F$$

The relevant quantity becomes now the probability density *P(x,t)* and the probability

$$p_0(t) = \int_{-\infty}^{0} \boldsymbol{P}(\boldsymbol{x},\boldsymbol{t})dx \quad and \quad p_1(t) = \int_{0}^{+\infty} \boldsymbol{P}(\boldsymbol{x},\boldsymbol{t})dx$$

Represent the probability for our switch to assume “0” or “1” logic states

This calls for a reconsideration of the equilibrium condition that now is:



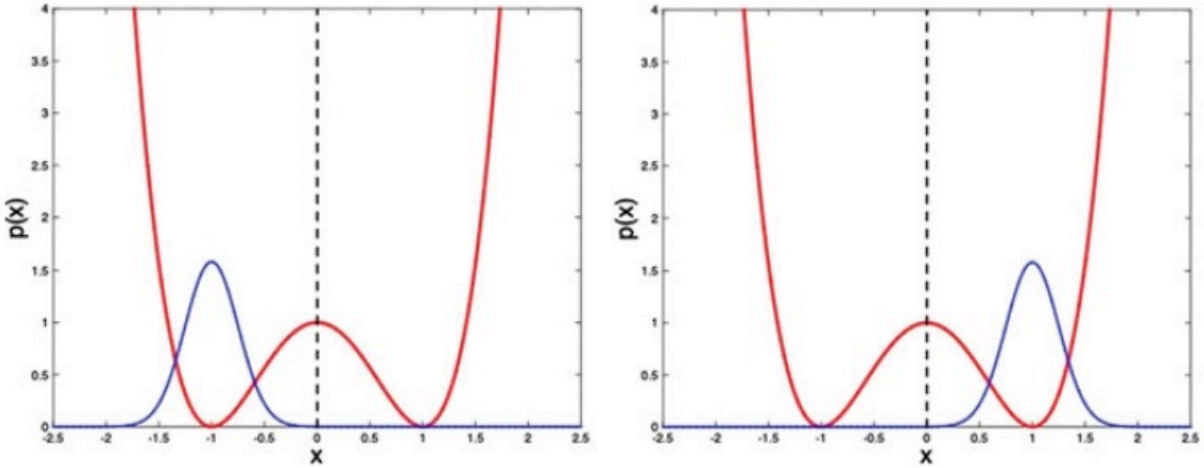
The Physics of realistic switches: the switch event

Based on these considerations we now define the switch event as the transition from an initial condition toward a final condition, where the initial condition is defined as <x> < 0 and the final condition is defined as <x> > 0. With the initial condition characterized by:

$$p_0(t) = \int_{-\infty}^{0} \boldsymbol{P}(\boldsymbol{x},\boldsymbol{t})dx \cong 1 \quad and \quad p_1(t) = \int_{0}^{+\infty} \boldsymbol{P}(\boldsymbol{x},\boldsymbol{t})dx \cong 0$$

and the final condition by:

$$p_0(t) = \int_{-\infty}^{0} \boldsymbol{P}(\boldsymbol{x},\boldsymbol{t})dx \cong 0 \quad and \quad p_1(t) = \int_{0}^{+\infty} \boldsymbol{P}(\boldsymbol{x},\boldsymbol{t})dx \cong 1$$



The Physics of realistic switches: Probability Density

The probability density $p(x, t)$ evolution can be monitored by computing it from the statistics of $x(t)$ or it can be described with a partial derivative differential equation, known as Fokker–Plank equation, after Adriaan Fokker (1887–1972) and Max Planck (1858–1947) that proposed it in 1914 and 1917 respectively, as a general diffusion equation.

The Fokker –Plank equation

$$\frac{\partial}{\partial t} p(x,t) = \left(-\frac{\partial}{\partial x} D^{(1)}(x) + \frac{\partial^2}{\partial x^2} D^{(2)}(x) \right) p(x,t)$$

The first term $D^{(1)}(x)$ is generally called the "drift" coefficient, and the second term $D^{(2)}(x)$ is called the "diffusion" coefficient. For a free particle, the drift coefficient is null, while for a particle that has no fluctuations the diffusion coefficient is null.

The Physics of realistic switches: Probability Density

In our case, we are faced with a constant diffusion coefficient (equilibrium thermal bath) and a drift term provided by the force associated with the confinement potential of the switch.

We have:

$$\frac{\partial}{\partial t} p(x,t) = \frac{\partial}{\partial x}\left(\frac{\partial V(x)}{\partial x} p(x,t)\right) + T \frac{\partial^2}{\partial x^2} p(x,t)$$

where $V(x)$ is the potential and T the temperature of the thermal bath. In general, the Fokker–Plank equation is a second-order partial differential equation, and in order to find its solutions, we need an initial condition $p(x, t_0)$ and boundary conditions for the space variable. Analytical solutions can be found in a limited number of cases. Among these there is the harmonic potential case, where the stationary solution is the already mentioned Gaussian probability density.

Stochastic thermodynamics (see Chap. 4 «The Physics of computing»)

The existence of fluctuations can be interpreted as due to the presence of a single dof ***x*** coupled to a thermal reserviour (large number N of dof).

At difference with the traditional Boltzmann approach, due to the presence of the confining potential V(x), not all the values of x(t) have the same probability to be realized.
Thus, in order to introduce a proper entropy, we should adopt W. Gibbs definition of thermodynamic entropy for microstates, identified by the index i and characterized by different probabilities p_i:

$$S = -k_B \sum_i p_i ln(p_i)$$

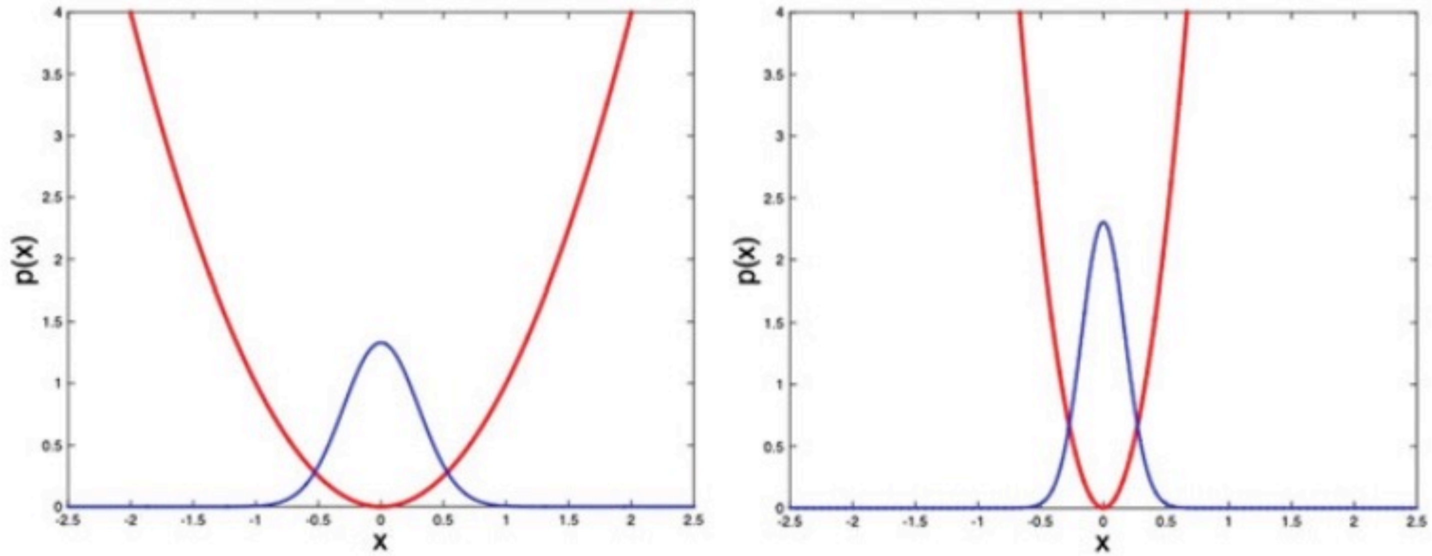
The available volume in the phase space is measured by the number of available microstates; thus, the sum is taken over all possible microstates, and each of them is weighted by the probability that our system actually realizes such a microstate.

The extension to a continuum of microstates was proposed by C. Shannon (1916–2001), few years later [5], as:

$$S = -k_B \int_{-\infty}^{+\infty} p(x) ln(p(x)) dx$$

Entropy and spread in probability density

By the moment that the probability of finding our switch variable $x(t)$ in the interval $[-\infty, +\infty]$ is clearly one, the spread of the probability density provides us an intuitive understanding of the entropy associated with different probability densities. As an example, in Fig. 4.1 we present two different arrangements for the probability density associated with a harmonic potential in the presence of Gaussian noise, at equilibrium. The temperature T, representative of the intensity of the fluctuations, is the same for the two arrangements, and the area below the probability density is one in both cases. However, the value of the potential $V(x)$ in Eq. (4.1) is different, being shallower in the left picture compared to the right one. The shallower is the potential, the spreader is the probability density, and the bigger is the associated entropy.

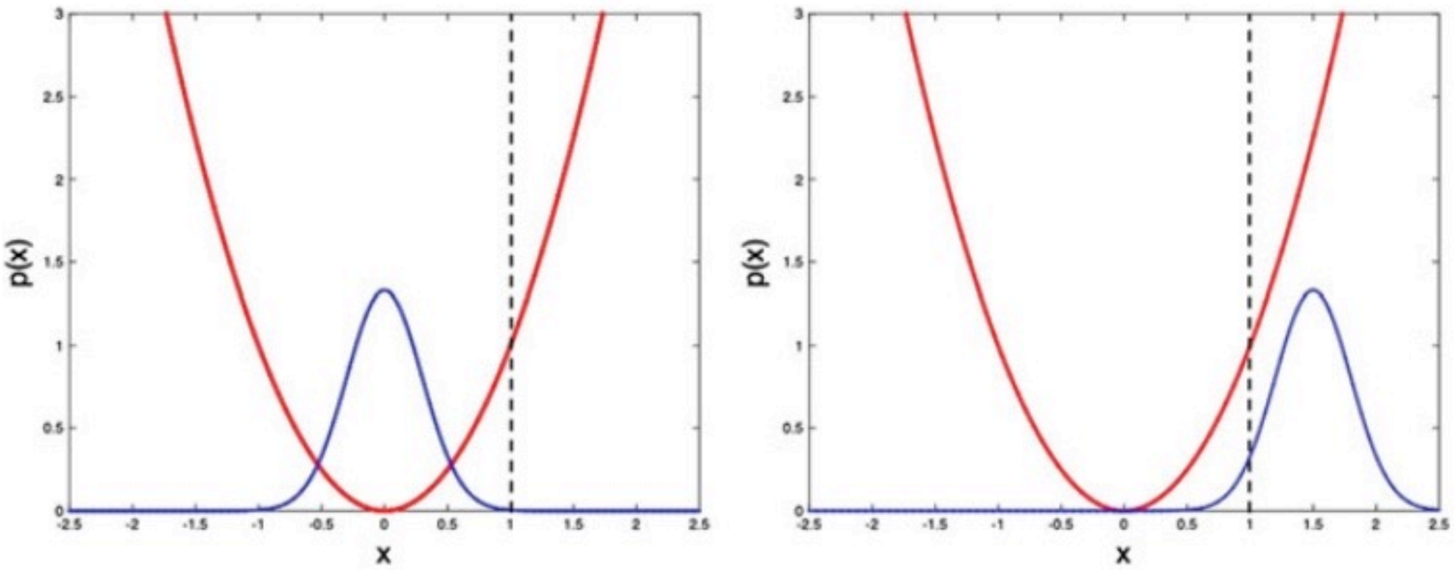


The Physics of realistic switches: the role of entropy

Now the question is: what happens to the entropy during the switch?

1) Combinational switches

As before, the switch event S_0 to S_1 can be generated by applying a deterministic constant force F_{ext} whose amplitude is properly chosen in order to satisfy the condition $p_1 > p_0$, for a given threshold x_{TH}. The switch event S_1 to S_0 is obtained once the force F_{ext} is removed.

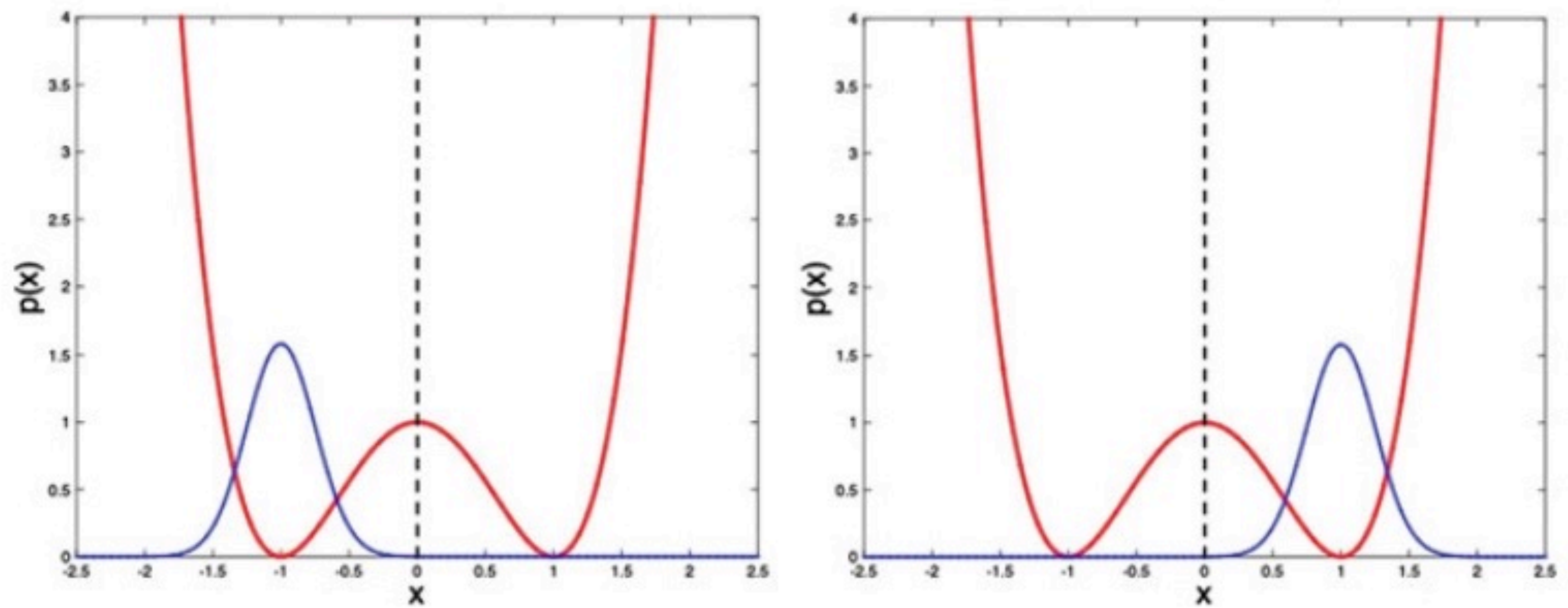


Same entropy initial and final

The Physics of realistic switches: the role of entropy

Now the question is: what happens to the entropy during the switch?

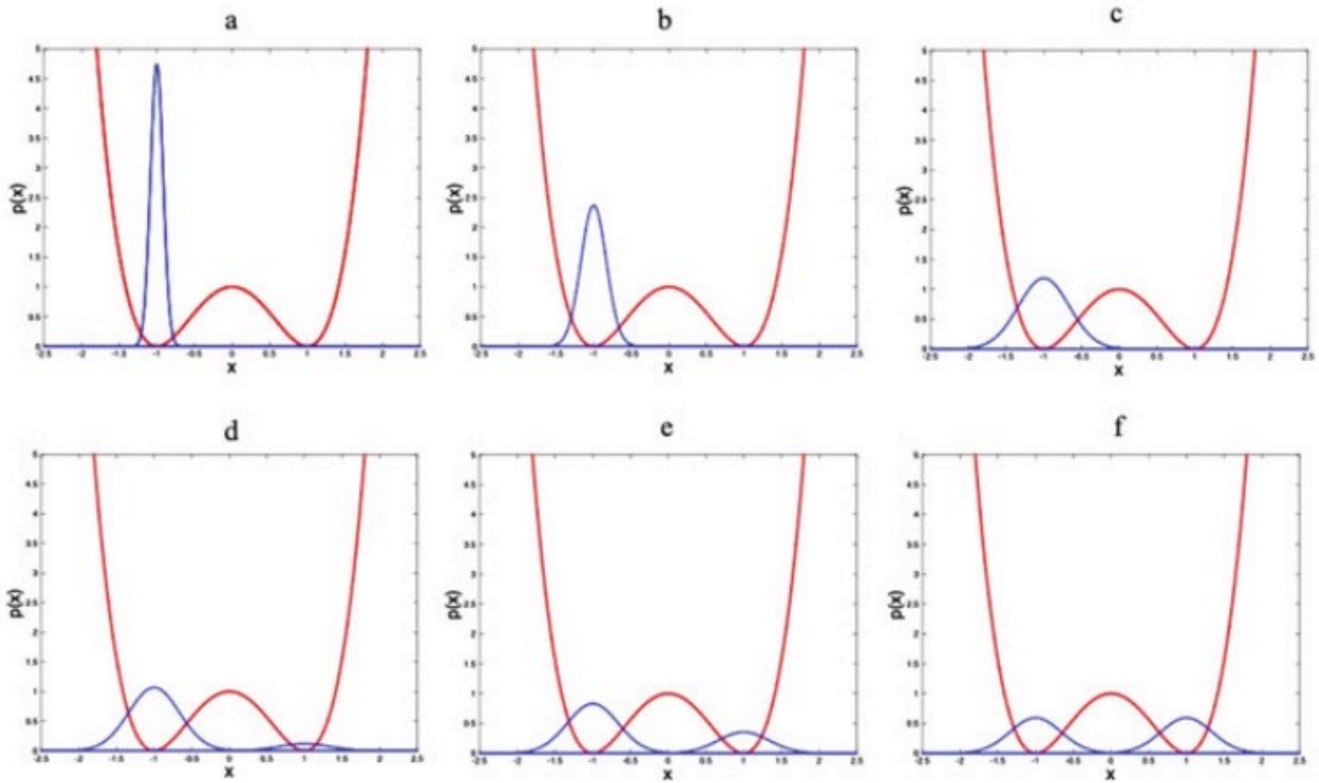
2) Sequential switches



Same entropy initial and final

Now the question is:

what happens to the entropy during the relaxation towards equilibrium?



Different entropy initial and final

We are now in position to discuss the enregy problem

Minimum energy required to operate the switch

(see chap 5 in “The Physics of Computing”)

1) **Combinational switches**
2) **Sequential switches**

Minimum energy required to operate the switch
(see chap 5 in "The Physics of Computing")

1) Combinational switches $\Delta U = L - Q$ Total energy variation during the switch

$L = F\,\Delta x > 0$ Potential energy of the system increases during the switch S0 to S1

Q = ? According to stochastic thermodynamics

$$Q = \left\langle \int_0^t -\frac{dV_e(x)}{dx}dx - \int_0^t m\dot{x}d\dot{x} \right\rangle$$

The second term is due to friction. It goes to zero if the speed goes to zero.
The first term can also be estimated as:

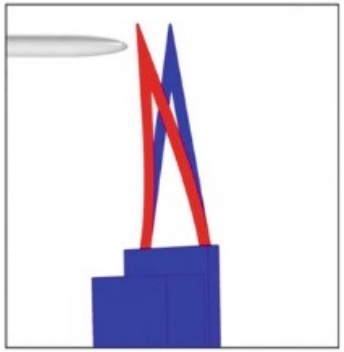
$$Q = T\,\Delta S$$

But $\Delta S = 0$ thus $Q = 0$

Conclusion: Some energy is required but no dissipation is necessary

Minimum energy required to operate the switch
(see chap 5 in "The Physics of Computing")

Experiment



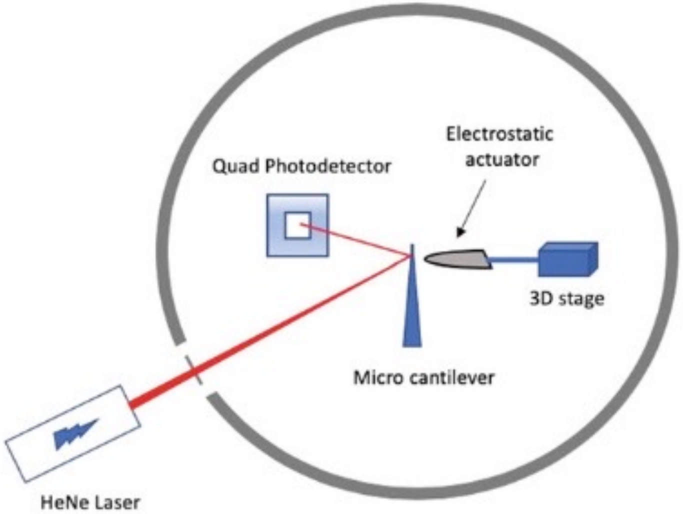
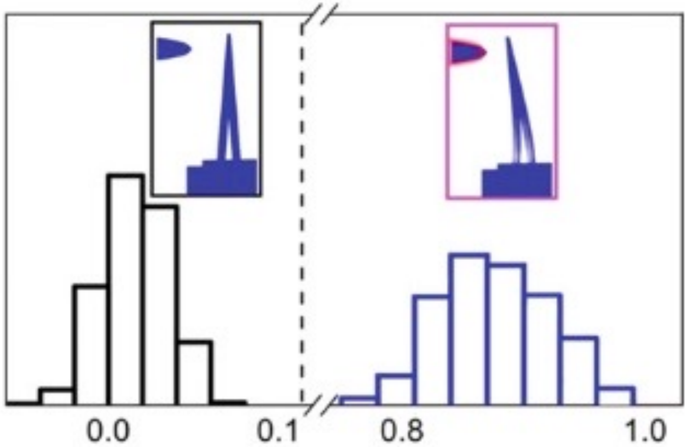
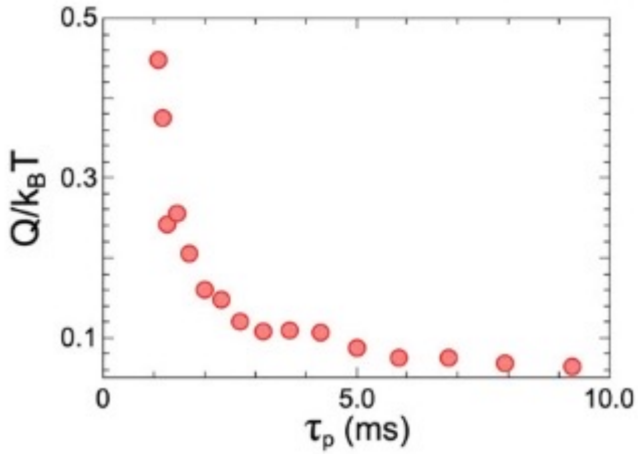


Fig. 5.3 Statistical distribution of the cantilever tip position without (left) and with (right) the electrostatic force. Position is expressed in nm. The vertical dashed line represents the threshold between S_0 and S_1 states





Minimum energy required to operate the switch
(see chap 5 in "The Physics of Computing")

1) Sequential switches

$\Delta U = L - Q$ Total energy variation during the switch

$L = 0$ Potential energy of the system does not increases during the switch S0 to S1

Q = ? According to stochastic thermodynamics

$$Q = \left\langle \int_0^t -\frac{dV_e(x)}{dx}dx - \int_0^t m\dot{x}d\dot{x} \right\rangle$$

The second term is due to friction. It goes to zero if the speed goes to zero.
The first term can also be estimated as:

$$Q = T\,\Delta S$$

But $\Delta S = 0$ thus $Q = 0$

Conclusion: No energy is required but…. Protocol matters

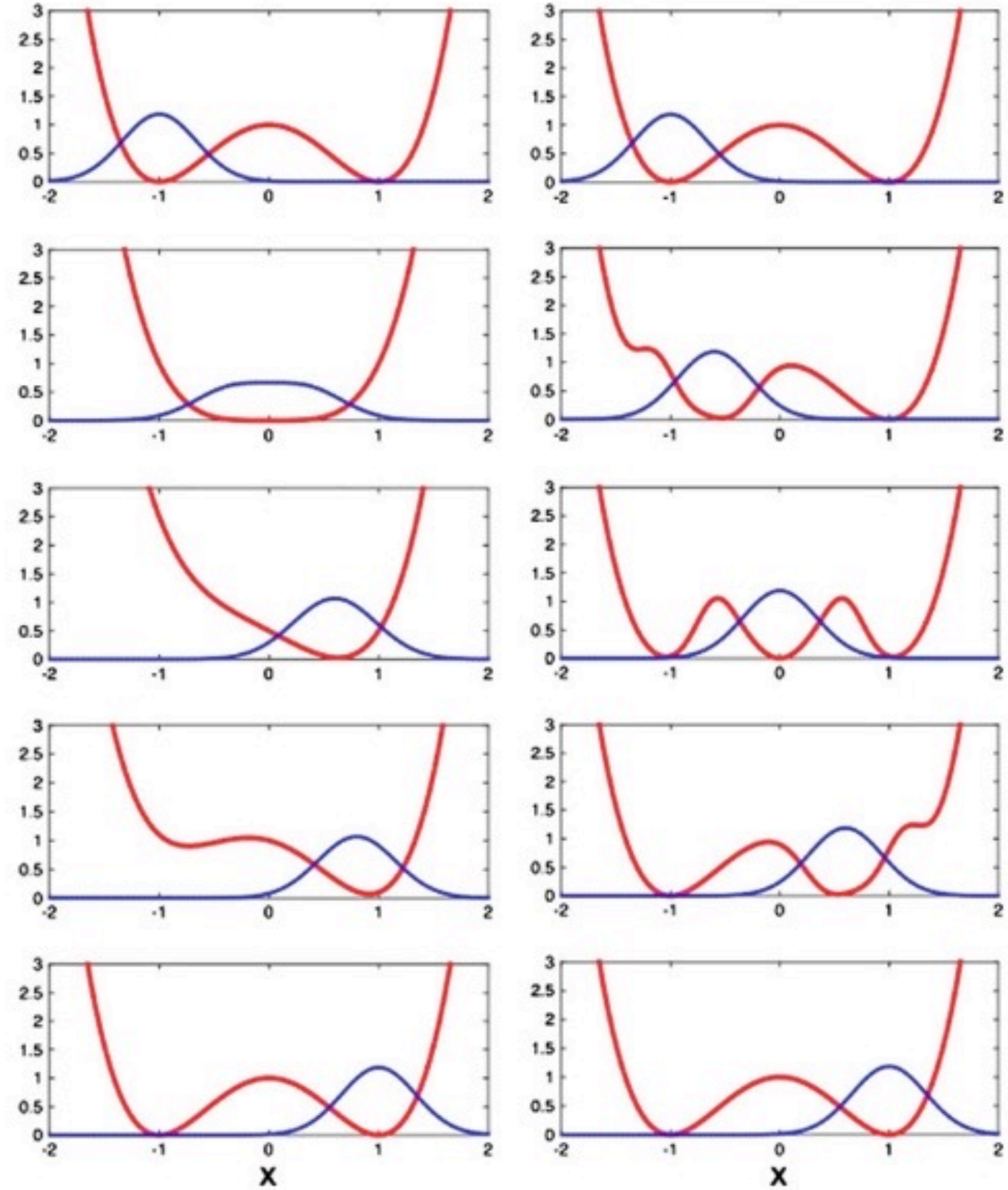
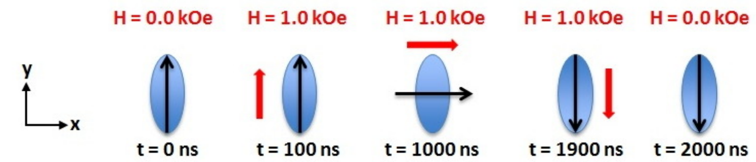
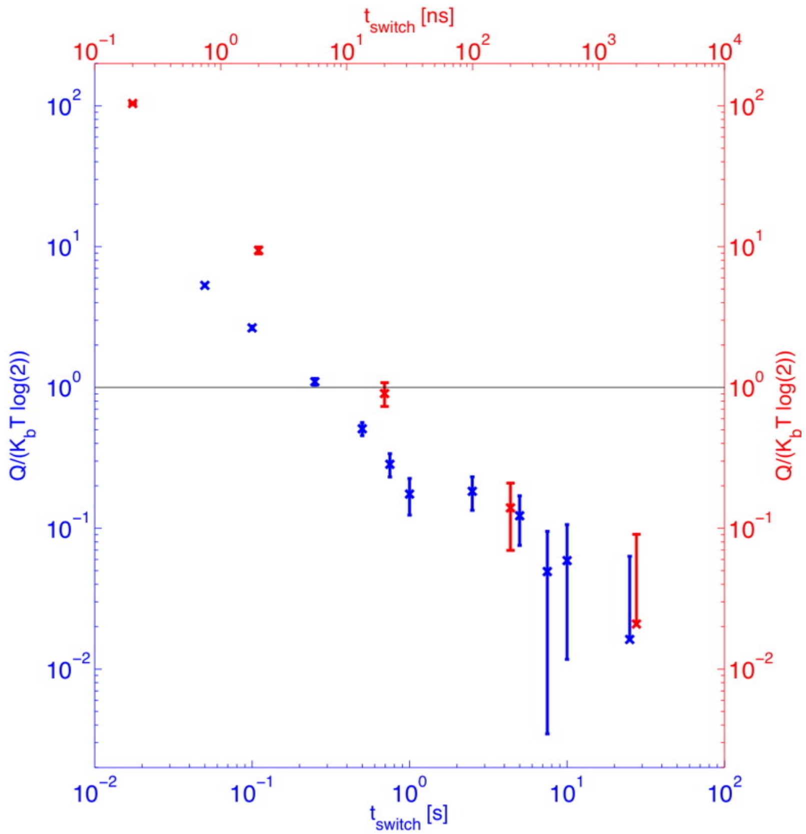


Fig. 5.6 Schematic representation of a switching procedure. Changes in the probability distribution (blue) and in the potential U(x) (red) due to the application of the external force, f(t). Steps 1–5, from top to bottom. Left: barrier drop *protocol.* Right: zero-power *protocol*.



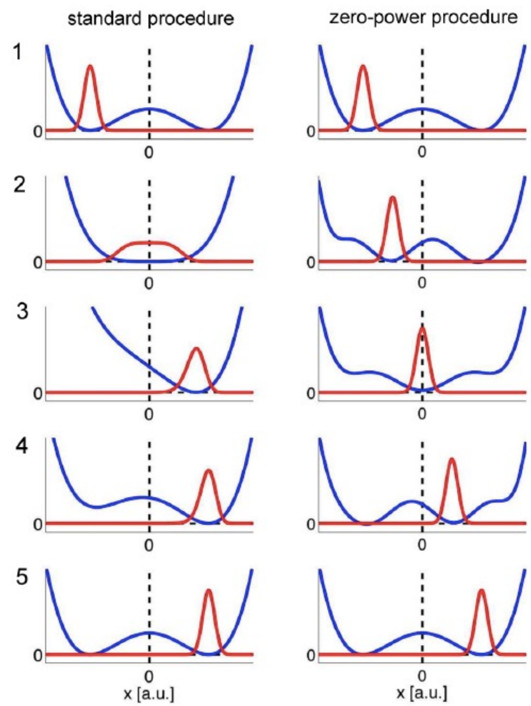


Magnetic nano dots

Single cylindrical element of permalloy (NiFe) with dimensions 50 x 50 x 5 nm³

Entropy changes

Entropy stays constant



More info available at www.landauer-project.eu

The Physics of realistic switches: the switch

In conclusion we have shown that, at least in principle, if the switch event is realized according to the following rules:

1) The total work performed on the system by the external force has to be zero (in a cycle).
2) The switch event has to proceed with a speed arbitrarily small in order to have arbitrarily small losses due to friction.
3) The system entropy **never decreases** during the switch event or the total change of the entropy is zero and the all process is realized through states of equilibrium.

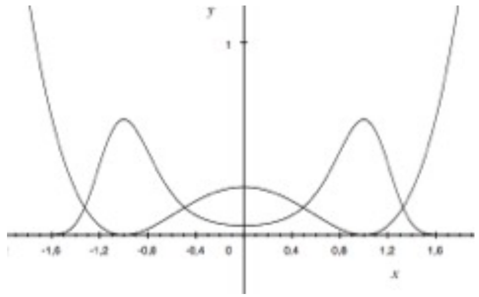
Then the switch event can be made by spending ZERO energy.

Thus, we can say that the computing activity, which is made by assembling switch events, can be made entirely by spending zero energy.

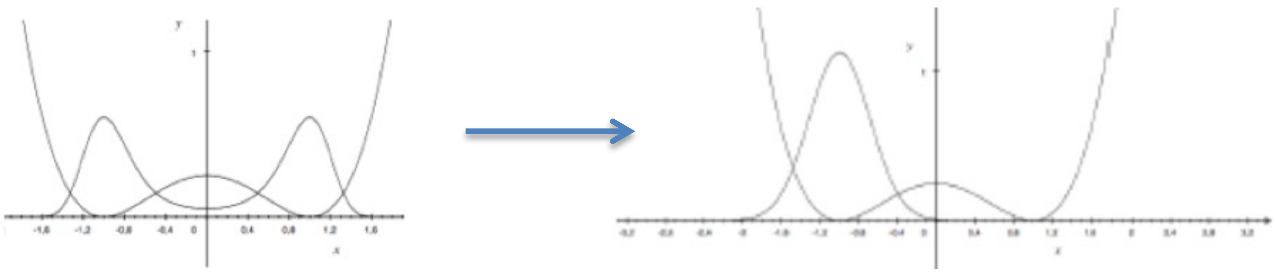
There is an exception for what concerns the memory use.

The Physics of realistic switches: the reset operation

But… let's suppose we start from an equilibrium condition



In this case if we want to write a memory bit or to use the switch we need to operate a **reset operation**



Is there a minimal cost for this operation?

THE LANDAUER LIMIT

The Landauer's principle states that erasing one bit of information (like in a resetting operation) comes unavoidably with a decrease in physical entropy and thus is accompanied by a minimal dissipation of energy equal to

$$Q = k_B\, T \ln 2$$

More technically this is the result of a change in entropy due to a change from a random state to a defined state.

Please note: this is the **minimum** energy required.

At room temperature $Q = 1.38\ 10^{-23}\ \ 300\ \ 0.69\ \ = 2{,}86\ 10^{-21}$ J

About $3\ 10^{-21}$ Joule

In conclusion

In conclusion we have shown that, at least in principle,

1) The switch event can be made by spending ZERO energy.

2) The reset event requires a minimum expenditure of energy. $Q = K_B T \ln(2)$